On the Impact of Decision Speed in Strategic Supply Chain Decentralization

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January 15, 2017

Abstract

The costs and benefits of vertical integration versus decentralization have been studied extensively. One strand of this large literature has highlighted the "strategic" benefits of decentralization. Strategic decentralization, or delegation, can benefit upstream firms by allowing them to commit to higher wholesale prices which then dampens downstream price competition. However, the strategic decentralization literature has ignored the possibility that while decentralization may affect the speed with which a supply chain makes decisions. In this paper, we focus on the impact of the decision speed on equilibrium strategies and supply chain structure.

1. Introduction

Consider a supply chain with an upstream firm ("manufacturer") and a downstream firm ("retailer"). In many settings, upstream and downstream firms in a supply chain work exclusively with each other. In these cases, competition should be thought as "chain-to-chain" or "channel-to-channel" competition. In other words, the upstream firm and downstream firm in one chain compete against not only the corresponding (upstream or downstream) firm in the other chain, but also the entire other chain. The performance of any firm in the chain will depend on the performance of the entire chain. However, it is not obvious that in such competition, the industry will be better off or a chain will be better off if one or all of the chains in the industry are integrated or coordinated.

Chain-to-chain or channel-to-channel competition has been addressed in the marketing and economics literatures. McGuire and Staelin (1983) analyze various retail distribution structures in the context of two competing manufacturers, each selling her products through an independent retailer. They show that a strategic reason for manufacturers to use intermediaries is that doing so shields themselves from possibly ruinous price competition. Bonanno and Vickers (1988), Coughlan (1985), Moorthy (1988), and Coughlan and Wernerfelt (1989) extend this research along various directions.

In this paper, we consider how decision speed affects supply chain structure. Decision speed refers to a supply chain's ability to make changes to pricing (or inventory) decisions. The supply chain literature has recognized the importance of the speed of operational decision making. For example, there is a literature that discusses the benefits of "quick response" and "fast fashion" strategies. However, the strategic role of decision speed has not been studied. We consider the impact of decision speed on the decision to decentralize or centralize a supply chain.

Our paper adds to the classic results on strategic decentralization. The strategic decentralization literature has shown that firms that compete on prices can benefit by decentralizing the price decision to a downstream agent. Basically, by committing (through decentralization) to a higher wholesale price, the firms can commit to dampening price competition through decentralization.

We show, however, that firms need to also consider the impact that decentralization has on decision speed. In the classical formulation, a decentralized firm can make decisions just as fast as a centralized firm. However, it is possible that decentralization affects the ability of the firm to respond to market signals. In fact, Richardson (1996) has argued that firms that have successfully implemented "quick response" and other similar strategies have usually accomplished this through vertical integration. This implies that decentralization compromises a firm's decision speed.

In strategic decentralization literature, decentralization is not accompanied by any impact on decision speed. This is arguably not true in practice. Decentralization could slow down the decision speed due to the fact that multiple parties in a decentralized supply chain need to coordinate on the strategies.

Thus, if decentralization is accompanied by slower decision speed, does this mean that this mitigates the impact of strategic decentralization? We show, somewhat surprisingly, that a slowing of decision speed can actually further benefit firms. In particular, we show that while decentralization that makes the firms "too slow" will destroy the strategic benefits of commitment, decentralization that is "moderately slow" will enhance the strategic value of decentralization. This implies that, when firms compete, they may not want to maximize their decision speed.

We consider an industry with two competing supply chains. Each chain has one manufacturer with one exclusive retailer. Two manufacturers produce differentiated but substitutable products. We examine three types of supply chain structures: both chains are integrated, both chains are decentralized, and one chain is integrated and the other is decentralized. In the mixed structure, we consider three sets of possible decision speeds.

2. The Model

The supply chain constituents and the demand system in our model follow Coughlan and Wernerfelt (1989). We describe the basic model setup in §2.1, and introduce supply chain speed and the resulting dynamics of the games in §2.2. For completeness, we present the key results from Coughlan and Wernerfelt (1989) in §2.3.

2.1 Supply Chains and Demand

There are two competing supply chains (or channels), indexed by $i = 1, 2$, with channel i supplying product i. Supply chain i has one manufacturer, referred to as manufacturer i, and one retailer, referred to as retailer i, who carries product i exclusively.

Demand is deterministic and price sensitive. For $i = 1, 2$, let $q_i(p_1, p_2)$ be the demand for product i when the prices of the two products are p_1 and p_2. Throughout the paper, the manufacturing cost is normalized to zero. Thus, channel i's profit is $\pi_i(p_1, p_2) = p_i \, q_i(p_1, p_2)$.

Assumption 1 *(i)* $q_i(p_1, p_2)$*,* $i = 1, 2$*, is continuously differentiable in* p_1 *and* p_2*.*
(ii) $\partial q_i/\partial p_i < 0, \quad |\partial q_i/\partial p_j| < |\partial q_i/\partial p_i|, \quad i, j = 1, 2, \; i \neq j.$
(iii) $(\partial q_1/\partial p_2)(\partial q_2/\partial p_1) > 0.$
(iv) The profit function $\pi_i(p_1, p_2)$ *is concave in* p_i *for any given* p_j*,* $i, j = 1, 2, \; i \neq j$*.*

Part (iii) implies that the cross-demand effects $\partial q_1/\partial p_2$ and $\partial q_2/\partial p_1$ have the same sign. The two products are substitutes if the cross-demand effect is positive, and they are complements if the cross-demand effect is negative.

When explicit analysis is difficult, we consider the following linear, symmetric demand system that is also used by Coughlan and Wernerfelt (1989):

$$p_i = A - Bq_i + Eq_j, \qquad i, j = 1, 2, \quad i \neq j, \tag{1}$$

or equivalently,

$$q_i(p_1, p_2) = \frac{A(B + E) - Bp_i - Ep_j}{B^2 - E^2}, \qquad i, j = 1, 2, \quad i \neq j. \tag{2}$$

The parameters satisfy $A, B > 0$ and $E \in (-B, B)$. Thus, all conditions in Assumption 1 are met. The products are complements if $E > 0$, whereas they are substitutes if $E < 0$.

2.2 Competition and Supply Chain Speed

The competition in the marketplace can be either price or quantity competition. The products are either substitutable or complementary. The main analysis of the paper focuses on price competition of substitutable products.

The competition between the two supply chains involves two stages. In the first stage, each supply chain chooses to be either decentralized or integrated. This gives the following possible industry structures: two integrated supply chains (denoted as II), two decentralized supply chains (denoted as DD), one integrated and one decentralized supply chain (denoted as ID, with possible variations in the decision speed, detailed soon after).

In the second stage, given the industry structure, the two supply chains compete in price, and a decentralized chain uses a contract to influence its retailer's decision. In this paper, we focus on a two-part contract, which consists of a wholesale price and a fixed fee. The wholesale price plays the role of coordinating the manufacturer and retailer to maximize channel profit, and the fixed payment allocates the supply chain profit. Thus, in determining the contract parameters, the manufacturer's objective is to maximize the channel profit.

When both chains are decentralized or integrated (II or DD), we assume the same decision speed for both supply chains. When one chain decentralizes and the other integrates, the supply chain speeds lead to various possible interactions between the chains. The dynamics of the games are detailed below.

1. **II**: The two integrated chains simultaneously decide retail price.
2. **DD**: The two manufacturers simultaneously choose the contract parameters, and then the two retailers simultaneously decide retail price.
3. Mixed supply chain structures. We consider three sets of game rules that differ in the timing of the decisions, reflecting the relative speed of the decentralized chain versus the integrated chain:
   a) **ID**: The decentralized chain decides contract parameters first, and then its decentralized retailer and the integrated chain simultaneously decides retail prices.
   b) **ID′**: The decentralized chain decides contract parameters first, and then the integrated chain determines price, and finally the decentralized retailer decides price.
   c) **ID″**: The decentralized chain determines contract parameters no sooner than the integrated chain determines price. The decentralized retailer decides price at the last.

Comparing the three timings for the mixed structure: in ID, decentralization does not slow down the retail price decision compared to the integrated chain. This is the setting considered by Coughlan and Wernerfelt (1989). The settings ID′ and ID″ are not considered in the literature but realistically capture the fact that decentralization may make the decision process slower than an integrated chain. Specifically, in ID′, the decentralized retailer's decision is delayed until after the integrated chain has made its decision; in ID″, decentralization significantly slows the decision process so that the contract is decided no sooner than the integrated chain's price decision.

Throughout the paper, we use p_i^{S}, w_i^{S}, q_i^{S}, π_i^{S}, $i = 1, 2$, to denote respectively the equilibrium retail price, wholesale price, sales quantity (or demand), and profit under channel structure S, where $\mathrm{S} \in \{\mathrm{M}, \mathrm{II}, \mathrm{DD}, \mathrm{ID}, \mathrm{ID}', \mathrm{ID}''\}$ and M standards for the monopoly setting. If the equilibrium is symmetric, we omit subscript i.

2.3 Summary of Existing Results

As a benchmark, consider a monopolist who owns both supply chains and aims to maximize the total industry profit $\sum_{i=1}^{2} p_i \, q_i(p_1, p_2)$. Under the linear demand system (1), it can be verified that the monopoly price and the profit of each channel are

$$p^{\mathrm{M}} = \frac{A}{2}, \qquad \pi^{\mathrm{M}} = \frac{A^2}{4(B - E)}.$$

Coughlan and Wernerfelt (1989) considered the price competition under the three basic structures (II, DD, and ID) and we summarize their results in the lemmas below. In II structure, supply chain i chooses price p_i to maximize its own profit (taking the other chain's price as given):

$$\max_{p_i} p_i\, q_i(p_1, p_2). \tag{3}$$

Lemma 1 *Under* II *structure and the demand system* (1)*, the equilibrium price is*

$$p^{\mathrm{II}} = \frac{A(B+E)}{2B+E} = \frac{A}{2}\Big(1 + \frac{E}{2B+E}\Big), \tag{4}$$

and the profit of each supply chain is

$$\pi^{\mathrm{II}} = \frac{A^2 B(B+E)}{(2B+E)^2(B-E)}. \tag{5}$$

In DD structure, the contract is parameterized by (w_i, f_i), where w_i is the wholesale price and f_i is the fixed fee that retailer i has to pay manufacturer i. Retailer i chooses price p_i to maximize its own profit:

$$\max_{p_i} (p_i - w_i)\, q_i(p_1, p_2) - f_i. \tag{6}$$

Note that the fixed fee f_i does not affect retailer's decision. For given wholesale prices w_1 and w_2, denote the Nash equilibrium retail prices as $p_i(w_1, w_2)$. Anticipating the equilibrium retail prices, manufacturer i chooses w_i to maximize its own chain's profit:

$$\max_{w_i} \; p_i(w_1, w_2)\, q_i\big(p_1(w_1, w_2), p_2(w_1, w_2)\big). \tag{7}$$

Lemma 2 *Under* DD *structure and the demand system* (1)*, the equilibrium wholesale price is*

$$w^{\mathrm{DD}} = \frac{AE^2(B+E)}{B(4B^2 + 2BE - E^2)},$$

the equilibrium retail price is

$$p^{\mathrm{DD}} = \frac{2AB(B+E)}{4B^2 + 2BE - E^2} = \frac{A}{2}\Big(1 + \frac{E(2B+E)}{4B^2 + 2BE - E^2}\Big), \tag{8}$$

and the profit of each supply chain is

$$\pi^{\mathrm{DD}} = \frac{2A^2 B(B+E)(2B^2 - E^2)}{(B-E)(4B^2 + 2BE - E^2)^2} = \pi^{\mathrm{II}}\Big(1 - \frac{E^3(4B+3E)}{(4B^2 + 2BE - E^2)^2}\Big). \tag{9}$$

The expressions in Lemma 2 are consistent with and simpler than those in Coughlan and Wernerfelt (1989, p. 236).

Consider the ID structure where we assume, without loss of generality, chain 1 is decentralized and chain 2 is integrated. After manufacturer 1 sets the wholesale price w_1, retailer 1 competes with chain 2 in setting retail prices. Retailer 1's problem is the same as in (6). Chain 2's problem is the same as in (3). Denote the Nash equilibrium retail prices as $p_i(w_1)$, $i = 1, 2$, for given wholesale price w_1.

In the first stage, manufacturer 1 chooses w_1 to maximize its own chain's profit:

$$\max_{w_1} \; p_1(w_1)\, q_1\big(p_1(w_1), p_2(w_1)\big). \tag{10}$$

Lemma 3 *Under* ID *structure and the demand system* (1)*, the equilibrium wholesale price is*

$$w^{\mathrm{ID}} = \frac{AE^2(2B-E)(B+E)}{4B^2(2B^2-E^2)}.$$

The equilibrium retail prices are

$$p_1^{\mathrm{ID}} = \frac{A}{2}\Big(1 + \frac{BE}{2B^2-E^2}\Big), \qquad p_2^{\mathrm{ID}} = \frac{A}{2}\Big(1 + \frac{E}{2B} - \frac{E^2}{2(2B^2-E^2)}\Big).$$

The equilibrium profits are

$$\begin{aligned}
\pi_1^{\mathrm{ID}} &= \frac{A^2(2B-E)^2(B+E)}{8B(B-E)(2B^2-E^2)} = \pi^{\mathrm{II}}\Big(1 + \frac{E^4}{8B^2(2B^2-E^2)}\Big),\\
\pi_2^{\mathrm{ID}} &= \frac{A^2(B+E)(4B^2-2BE-E^2)^2}{16B(B-E)(2B^2-E^2)^2} = \pi^{\mathrm{DD}}\Big(1 - \frac{E^4(16B^4-8B^2E^2-E^4)}{32B^2(2B^2-E^2)^3}\Big).
\end{aligned}$$

Note that Lemma 3 simplifies the expressions in Coughlan and Wernerfelt (1989, p. 235).

Based on the above lemmas, we have the following order of profits. For substitutes ($E < 0$):

$$\pi^{\mathrm{II}} < \pi_1^{\mathrm{ID}} < \pi_2^{\mathrm{ID}} < \pi^{\mathrm{DD}} < \pi^M \qquad \text{and} \qquad 2\pi^{\mathrm{II}} < (\pi_1^{\mathrm{ID}} + \pi_2^{\mathrm{ID}}) < 2\pi^{\mathrm{DD}} < 2\pi^M.$$

For complements ($E > 0$):

$$\pi_2^{\mathrm{ID}} < \pi^{\mathrm{DD}} < \pi^{\mathrm{II}} < \pi_1^{\mathrm{ID}} < \pi^M \qquad \text{and} \qquad 2\pi^{\mathrm{DD}} < (\pi_1^{\mathrm{ID}} + \pi_2^{\mathrm{ID}}) < 2\pi^{\mathrm{II}} < 2\pi^M.$$

In either case, $\pi^{\mathrm{II}} < \pi_1^{\mathrm{ID}}$ and $\pi_2^{\mathrm{ID}} < \pi^{\mathrm{DD}}$ together imply that decentralization is the dominating strategy, and that the DD is the unique equilibrium industry structure.

Lemma 4 DD *is the unique equilibrium regardless products are substitutes or complements. For substitutes (complements),* DD *is the duopoly structure that the two chains' combined profit is the highest (lowest).*

3. Impact of Decision Speed on Price Competition and Supply Chain Structure

As discussed in §2.2, supply chain speed affects the dynamics of the game, which fundamentally changes the equilibrium of the competition. In this section, we analyze and compare the equilibria under ID, ID′, and ID″, and determine the equilibrium choice of supply chain structure. We assume that, when supply chains choose their structures in the very first stage, the speed is exogenously given. That is, the supply chains cannot choose among ID, ID$'$, and ID$''$. Rather, both chains know that if the mixed structure is chosen, the rule of the game follows either ID or ID$'$ or ID$''$. We will relax this assumption in §4 where we allow supply chains to choose their speed and structure.

Throughout our analysis of ID, ID$'$, and ID$''$ structures, we assume chain 1 is decentralized and chain 2 is integrated.

3.1 Decision Speed and Stackelberg Leadership

To understand the effect of decision speed, we first consider a setting similar to II structure, but one chain is the Stackelberg leader in deciding its price. From this point onward, Stackelberg leader (follower) refers to the leader (follower) in the Stackelberg game of two integrated chains.

Without loss of generality, let chain 1 be the leader and chain 2 be the follower. Let p_1^{lead} and p_2^{follow} denote the equilibrium prices of the Stackelberg game. The follower's problem is $\max_{p_2} p_2\, q_2(p_1, p_2)$, the same problem as in the II structure (see (3)). The follower's best response to the leader's p_1 is denoted as $p_2^*(p_1)$. Then, the leader's problem is

$$\max_{p_1} p_1\, q_1(p_1, p_2^*(p_1)). \tag{11}$$

The optimal price to the above problem is p_1^{lead}. The follower's best response is $p_2^{\text{follow}} = p_2^*(p^{\text{lead}})$.

The following proposition identifies the relation between the ID structure and the Stackelberg game of the two integrated chains.

Proposition 1 *In* ID *structure, the decentralized chain's equilibrium retail price and profit are the same as those of the Stackelberg leader, whereas the integrated chain's equilibrium retail price and profit are the same as those of the Stackelberg follower.*

Proposition 1 implies that strategic decentralization is equivalent to gaining the leadership in the game, provided that decentralization does not delay the retail pricing decision.

Example 1 Under the demand system (2), supply chain 2's problem is

$$\max_{p_2} p_2\, q_2(p_1, p_2) = \frac{-Bp_2^2 + p_2\big(A(B+E) - Ep_1\big)}{B^2 - E^2} \tag{12}$$

The best p_2 in response to p_1 is

$$p_2^*(p_1) = \frac{A(B+E) - Ep_1}{2B}. \tag{13}$$

Chain 1 sets p_1 in anticipation of chain 2's best response:

$$\max_{p_1}\; p_1\, q_1(p_1, p_2^*(p_1)) = p_1\, \frac{A(B+E) - Bp_1 - E\frac{A(B+E)-Ep_1}{2B}}{B^2 - E^2}$$

The optimal p_1 from the above optimization is:

$$p^{\text{lead}} = \frac{A(B+E)(2B-E)}{2(2B^2-E^2)} = \frac{A}{2}\Big(1+\frac{BE}{2B^2-E^2}\Big). \tag{14}$$

The best response of chain 2 is:

$$p^{\text{follow}} = \frac{A(B+E)-Ep^{\text{lead}}}{2B} = \frac{A}{2}\Big(1+\frac{E}{2B}-\frac{E^2}{2(2B^2-E^2)}\Big) \tag{15}$$

We can derive that the supply chains' profits are:

$$\pi^{\text{lead}} = \frac{A^2(B+E)(2B-E)^2}{8B(B-E)(2B^2-E^2)} = \pi^{\text{II}}\Big(1+\frac{E^4}{8B^2(2B^2-E^2)}\Big), \tag{16}$$

$$\pi^{\text{follow}} = \frac{A^2(B+E)(4B^2-2BE-E^2)^2}{16B(B-E)(2B^2-E^2)^2} = \pi^{\text{DD}}\Big(1-\frac{E^4(16B^4-8B^2E^2-E^4)}{32B^2(2B^2-E^2)^3}\Big). \tag{17}$$

Comparing the above results with Lemma 3, we have

$$p_1^{\text{ID}} = p^{\text{lead}}, \qquad p_2^{\text{ID}} = p^{\text{follow}}, \qquad \pi_1^{\text{ID}} = \pi^{\text{lead}}, \qquad \pi_2^{\text{ID}} = \pi^{\text{follow}}. \qquad \blacksquare$$

Next, we solve for the equilibrium under ID$''$, in which decentralization significantly slows down the supply chain speed in making decisions. As a result, the decentralized manufacturer determines contract parameters no sooner than the integrated chain determines price.

Proposition 2 *In* ID$''$ *structure, the integrated chain's equilibrium retail price and profit are the same as those of the Stackelberg leader, whereas the decentralized chain's equilibrium retail price and profit are the same as those of the Stackelberg follower. Furthermore, the decentralized manufacturer sets the wholesale price equal to its marginal cost.*

Proposition 2 implies that if strategic decentralization slows down the decision such that the contract parameters are decided simultaneously as or later than the integrated chain determines price, then strategic decentralization is equivalent to committing being the follower in retail pricing. Even if the decentralized manufacturer chooses the wholesale price simultaneously as the integrated chain decides the retail price, the integrated chain still has leadership in the game. This is because the decentralized chain is unable to commit the wholesale price before the integrated chain moves.

Example 2 (ID$''$ under linear demand) Under the demand system (2), given w_1 and p_2, the decentralized retailer's problem is the same as in (6):

$$\max_{p_1}\,(p_1-w_1)q_1(p_1,p_2) = (p_1-w_1)\frac{-Bp_2+\big(A(B+E)-Ep_j\big)}{B^2-E^2}$$

The best response function is

$$p_1^*(w_1,p_2) = \frac{A(B+E)+Bw_1-Ep_2}{2B}. \tag{18}$$

In the Nash game, the decentralized manufacturer's problem is

$$\max_{w_1} \ p_1^*(w_1, p_2) q_1\big(p_1^*(w_1, p_2), p_2\big) = \frac{A(B+E) + Bw_1 - Ep_2}{2B}\Big(\frac{A(B+E) - B\frac{A(B+E)+Bw_1-Ep_2}{2B} - Ep_2}{B^2-E^2}\Big)$$
$$= \frac{(A(B+E) - Ep_2)^2 - B^2 w_1^2}{4B(B^2-E^2)}$$

Thus, the optimal wholesale price is $w^{\mathrm{ID}''} = 0$ regardless of p_2.

In response to $w^{\mathrm{ID}''} = 0$, the integrated chain's problem is the same as the problem of the Stackelberg leader in the II structure. ∎

Thus far, we show the benefit of strategic decentralization—early commitment of contract parameters to gain strategic advantage—can be realized when decentralization does not slow down the supply chain speed in deciding the retail price. This benefit no longer exists when decentralization slows down the decision to such an extent that early commitment of contract parameters becomes infeasible, as in the ID$''$ structure.

3.2 Competition Under ID$'$ Structure

In ID$'$ structure, the decentralized chain 1 decides wholesale price w_1 first, the integrated chain 2 then decides p_2, and finally retailer 1 decides retail price p_1. Intuitively, because the speed of the decentralized chain in ID$'$ is between that in ID and ID$''$ structures, we expect that the equilibrium prices and profits in ID$'$ are between that in ID and ID$''$, but the results are counter-intuitive.

Because the game under ID$'$ structure involves three stages, analyzing and comparing the equilibria under the general demand system becomes intractable. Thus, we analyze ID$'$ structure using the linear demand system in (2).

For notational simplicity, we let $p^{\mathrm{D}} \equiv p_1^{\mathrm{ID}'}$, $p^{\mathrm{I}} \equiv p_2^{\mathrm{ID}'}$, $\pi^{\mathrm{D}} \equiv \pi_1^{\mathrm{ID}'}$, $\pi^{\mathrm{I}} \equiv \pi_2^{\mathrm{ID}'}$.

Proposition 3 *In* ID$'$ *structure and the demand system in* (2), *in equilibrium,*

(i) The decentralized manufacturer sets the wholesale price

$$w^{\mathrm{ID}'} = \frac{AE^2(B+E)(4B^2 - 2BE - E^2)}{B(4B^2-E^2)(4B^2-3E^2)}, \tag{19}$$

(ii) The integrated chain sets the retail price

$$p^{\mathrm{I}} = \frac{A}{2}\Big(1 + \frac{E(8B^3 - 6BE^2 - E^3)}{(4B^2-E^2)(4B^2-3E^2)}\Big), \tag{20}$$

(iii) The decentralized retailer sets the retail price

$$p^{\mathrm{D}} = \frac{A}{2}\Big(1 + \frac{E(2B^2-E^2)}{B(4B^2-3E^2)}\Big). \tag{21}$$

(iv) The equilibrium profits of the decentralized chain and integrated chain are respectively

$$\pi^{\mathrm{D}} = \frac{A^2(B+E)(4B^2-2BE-E^2)^2}{4B(B-E)(4B^2-E^2)(4B^2-3E^2)}, \tag{22}$$

$$\pi^{\mathrm{I}} = \frac{A^2(B+E)(2B^2-E^2)(8B^3-4B^2E-4BE^2+E^3)^2}{2B(B-E)(4B^2-E^2)^2(4B^2-3E^2)^2}. \tag{23}$$

Combining the results in Propositions 1-3, Lemmas 1-4, we can order the prices, channel profits, and industry profits, as described in the following corollary.

Corollary 4 *Under the demand system in* (2) *and substitutable products ($E < 0$),*

(i) $w^{\mathrm{ID}'} > \max\{w^{\mathrm{DD}}, w^{\mathrm{ID}}\} > w^{\mathrm{ID}''} = 0$,

(ii) $p^{\mathrm{II}} < p^{\mathrm{follow}} < p^{\mathrm{lead}} < p^{\mathrm{DD}} < p^{\mathrm{I}} < p^{\mathrm{D}} < p^{\mathrm{M}}$,

(iii) $\pi^{\mathrm{II}} < \pi^{\mathrm{lead}} < \pi^{\mathrm{follow}} < \pi^{\mathrm{DD}} < \pi^{\mathrm{D}} < \pi^{\mathrm{I}} < \pi^{\mathrm{M}}$,

(iv) $2\pi^{\mathrm{II}} < (\pi^{\mathrm{lead}} + \pi^{\mathrm{follow}}) < 2\pi^{\mathrm{DD}} < (\pi^{\mathrm{D}} + \pi^{\mathrm{I}}) < 2\pi^{\mathrm{M}}$.

Our prior expectation is that the faster the decentralized chain, the higher wholesale price it can commit, but Corollary 4 part (i) shows counter-intuitively that the equilibrium wholesale price under ID$'$ is the highest among all of the duopoly structures considered. Consequently, the retail prices p^{I} and p^{D} under ID$'$ structure are also the highest among all of the duopoly structures considered, stated in part (ii). Furthermore, both supply chains earn higher profits under ID$'$ structure than under any other structure, confirmed in part (iii). (Recall that under ID or ID$''$, the supply chains earn either π^{lead} or π^{follow}.) In part (iv), the industry total profit under ID and ID$''$ structures, $\pi^{\mathrm{lead}} + \pi^{\mathrm{follow}}$, is below the industry profit under DD structure, which is the unique equilibrium supply chain structure found in Coughlan and Wernerfelt (1989). However, the industry profit under ID$'$ structure is even higher than that under DD structure. Thus, the speed of the supply chains may lead to different equilibrium supply chain structure, which we explore below.

3.3 Equilibrium Supply Chain Structures

In this section, we assume that the supply chain speeds under decentralization and integration are exogenously given. That is, supply chains cannot choose among ID, ID$'$, and ID$''$. For each given speed level, we derive the equilibrium of the supply chain structure game where the supply chains simultaneously choose whether to decentralize or integrate.

Proposition 5 *Under the demand system in* (2) *and substitutable products ($E < 0$),*

(i) (Coughlan and Wernerfelt 1989) If the decentralized chain decides retail price no later than the integrated chain decides its price (i.e., mixed structure is ID*), then* DD *is the unique equilibrium;*

(ii) If the decentralized chain decides contract before the integrated chain decides price, followed by the decentralized retailer deciding price (i.e., mixed structure is ID$'$*), then* ID$'$ *is the unique equilibrium.*

(iii) If the decentralized chain decides contract no sooner than the integrated chain decides price (i.e., mixed structure is ID$''$*), then* DD *is the unique equilibrium.*

Proof. The results follow directly from Corollary 4 (iii): $\pi^{\mathrm{II}} < \pi^{\mathrm{lead}} < \pi^{\mathrm{follow}} < \pi^{\mathrm{DD}} < \pi^{\mathrm{D}} < \pi^{\mathrm{I}}$. ∎

When the speed of a decentralized chain is either fast (ID) or slow (ID$''$), the unique equilibrium is both supply chain choosing to be decentralized (DD). However, when the speed of a decentralized chain is medium (ID$'$), then ID$'$ is the unique equilibrium supply chain structure, bringing both chains the highest profit.

Intuitively, sequential moves bring an advantage over simultaneous moves in the price competition for substitutable products. For example, comparing simultaneous move and sequential move when both chains are integrated, we have $p^{\mathrm{II}} < p^{\mathrm{follow}} < p^{\mathrm{lead}}$ and $\pi^{\mathrm{II}} < \pi^{\mathrm{lead}} < \pi^{\mathrm{follow}}$. This intuition helps explain why ID$'$ leads to a higher profit than DD: The game in DD structure involves simultaneous moves at both wholesale and retail level, whereas ID$'$ structure involves only one move of one supply chain at a time, and the moves are staggered. The sequential-move nature of ID$'$ structure renders it more profitable than DD. This result significantly departs from the literature and reveals the importance of the supply chain speed.

4. Supply Chain Speed Choice

We relax the assumption that the supply chain speed is exogenously given.

4.1 One Speed Level for Given Structure

We assume that there is only one speed level for a integration chain and one speed level for a decentralized chain. The speed is a decision variable and can be chosen at the same time as choosing the supply chain structure. We refer to the choice as speed-structure choice. The possible supply structures include II, DD, ID, ID$'$, and ID$''$. In particular, if both chain choose to integrated or decentralize, the single-speed assumption implies that the two chains have the same speed, resulting in either II or DD structure.

Proposition 6 *Under the demand system in* (2) *and substitutable products ($E < 0$), suppose the supply chains have one speed level under either decentralization or integration structure. Then, the unique equilibrium supply chain structure is the mixed structure* ID$'$.

Proof. From Corollary 4 (iii), $\pi^{\mathrm{II}} < \pi^{\mathrm{lead}} < \pi^{\mathrm{follow}} < \pi^{\mathrm{DD}} < \pi^{\mathrm{D}} < \pi^{\mathrm{I}}$. If one chain is integrated, the unique optimal speed-structure choice of the other chain is to decentralize and form ID$'$ structure. Similarly, if one chain is decentralized, the unique optimal speed-structure choice of the other chain is to integrated and form ID$'$ structure. ∎

4.2 Multiple Speed Levels for Given Structure

Finally, we allow different speed levels under the same structural choice. The case of two integrated chains having different speeds is exactly the Stackelberg game considered earlier. We focus on analyzing the new speed-structure choice where the two decentralized chains have different speeds.

Consider the following sequence of decisions: First, chain 2 decides contract; second, chain 1 decides contract; third, retailer 2 decides price; finally, retailer 1 decides price. That is, the sequence of decision is w_2, w_1, p_2, p_1. We label this game as DD$'$ structure. In this sequence of actions, chain 2 is the leader in both stages, and chain 1 is the follower in both stages. Other sequences are possible, but they are equivalent to other structures. For example, the sequence w_1, w_2, p_2, p_1 is equivalent to ID$'$ with chain 2 being an integrated chain; the sequence w_1, p_1, w_2, p_2 is equivalent to a Stackelberg game between two integrated chains.

Once the speed level is chosen, we assume that both supply chains commit to the decision sequence.

Proposition 7 *In* DD$'$ *structure and the demand system in* (2)*, in equilibrium,*

(i) The manufacturers set the wholesale prices

$$w_2^{\mathrm{DD}'} = \frac{AE^4(B+E)(8B^3-4B^2E-4BE^2+E^3)}{4(2B^2-E^2)^2(8B^4-8B^2E^2+E^4)}$$
$$w_1^{\mathrm{DD}'} = \frac{A(2B-E)E^2(B+E)(8B^3-6BE^2-E^3)}{4B(2B^2-E^2)(8B^4-8B^2E^2+E^4)}$$

(ii) The retailers set the retail prices

$$p^{\mathrm{DDlead}} \equiv p_2^{\mathrm{DD}'} = \frac{1}{2}\Big(A + \frac{AE(4B^3-3BE^2)}{8B^4-8B^2E^2+E^4}\Big)$$
$$p^{\mathrm{DDfollow}} \equiv p_1^{\mathrm{DD}'} = \frac{A(2B-E)^2(B+E)(2B+E)(8B^3-6BE^2-E^3)}{8B(2B^2-E^2)(8B^4-8B^2E^2+E^4)}$$

(iii) The equilibrium profits are

$$\pi^{\mathrm{DDlead}} \equiv \pi_2^{\mathrm{DD}'} = \frac{A^2(B+E)(8B^3-4B^2E-4BE^2+E^3)^2}{16B(B-E)(2B^2-E^2)(8B^4-8B^2E^2+E^4)}$$
$$\pi^{\mathrm{DDfollow}} \equiv \pi_1^{\mathrm{DD}'} = \frac{A^2(2B-E)^3(B+E)(2B+E)(4B^2-3E^2)(8B^3-6BE^2-E^3)^2}{64B(B-E)(2B^2-E^2)^2(8B^4-8B^2E^2+E^4)^2}$$

Corollary 8 *Under the linear demand system in* (2) *and substitutable products* ($E<0$)*,*

(i) $p^{\mathrm{II}} < p^{\mathrm{follow}} < p^{\mathrm{lead}} < p^{\mathrm{DD}} < p^{\mathrm{I}} < p^{\mathrm{D}} < p^{\mathrm{DDfollow}} < p^{\mathrm{DDlead}} < p^{\mathrm{M}}$,

(ii) $\pi^{\mathrm{II}} < \pi^{\mathrm{lead}} < \pi^{\mathrm{follow}} < \pi^{\mathrm{DD}} < \pi^{\mathrm{D}} < \pi^{\mathrm{I}} < \pi^{\mathrm{DDlead}} < \pi^{\mathrm{DDfollow}} < \pi^{\mathrm{M}}$,

(iii) $2\pi^{\mathrm{II}} < (\pi^{\mathrm{lead}} + \pi^{\mathrm{follow}}) < 2\pi^{\mathrm{DD}} < (\pi^{\mathrm{D}} + \pi^{\mathrm{I}}) < (\pi^{\mathrm{DDlead}} + \pi^{\mathrm{DDfollow}}) < 2\pi^{\mathrm{M}}$.

Proposition 9 *Under the demand system in* (2) *and substitutable products* ($E<0$)*, suppose the supply chains can choose different speed levels under either decentralization or integration structure. Then, the unique equilibrium supply chain structure is* DD$'$.

Proof. From Corollary 8 (ii), we see that if one chain is decentralized, the unique optimal speed-structure choice of the other chain is to decentralize but with a slower speed to form DD$'$ structure. ∎

Corollary 8 shows that DD$'$ structure leads to a higher profit for each supply chain than all other duopoly structures. This reinforces the insight that staggered moves in the game allows both supply chains to raise their prices and reap a higher profit.

Proposition 9 confirms that DD$'$ is the equilibrium supply chain structure.

However, there is a catch when supply chains can freely choose the speed. Because the follower in the DD$'$ structure earns a higher profit, both supply chains prefer to slow down their decisions. As a result, in the equilibrium, both chains may become very slow.

5. Complementary Products

The results for complementary products also depart from the classic results.

Corollary 10 *Under the linear demand system in* (2) *and complementary products* ($E > 0$),

(i) $p^{\mathrm{M}} < p^{\mathrm{follow}} < p^{\mathrm{II}} < p^{\mathrm{I}} < p^{\mathrm{DD}} < p^{\mathrm{D}} < p^{\mathrm{lead}}$,

(ii) $\max\{\pi^{\mathrm{follow}}, \pi^{\mathrm{I}}\} < \pi^{\mathrm{DD}} < \pi^{\mathrm{D}} < \pi^{\mathrm{II}} < \pi^{\mathrm{lead}} < \pi^{\mathrm{M}}$,

(iii) $(\pi^{\mathrm{D}} + \pi^{\mathrm{I}}) < 2\pi^{\mathrm{DD}} < (\pi^{\mathrm{lead}} + \pi^{\mathrm{follow}}) < 2\pi^{\mathrm{II}} < 2\pi^{\mathrm{M}}$.

Proposition 11 (No Speed Choice) *Under the linear demand system in* (2) *and complementary products* ($E > 0$),

(i) ID, *then* DD *is the unique equilibrium;*

(ii) ID$'$, *then both* II *and* DD *are equilibria;*

(iii) ID$''$, *then* II *is the unique equilibrium.*

For complementary products, Corollary 10 (iii) shows that the order of duopoly profits reverses compared to the substitutable products. In particular, II is the most profitable duopoly structure. Intuitively, decentralization and sequential moves tend to raise prices, which is not desirable for profiting from a market with complementary products.

In Proposition 11(i) shows the prisoner's dilemma: Both chains have incentives to decentralize, and end up with the DD structure as an equilibrium. This is also shown in Coughlan and Wernerfelt (1989).

Interestingly, the prisoner's dilemma no longer exists when decentralization slows down the decision process relative to the integrated chain. If decentralization slightly slows down the decision so that ID$'$ is in consideration, then both II and DD can be equilibria. If decentralization significantly slows down the decision so that ID$''$ is in consideration, then II emerges as the unique equilibrium.

Proposition 12 (Single Speed Choice) *Under the demand system in* (2) *and complementary products* ($E > 0$)*, there does not exist a pure-strategy equilibrium structure.*

Proof: If chain 1 integrates, chain 2 will choose to decentralize with fast speed (forming ID to earn π^{lead}). If chain 2 decentralizes, chain 1 will choose to integrate with fast speed (forming ID$''$ to earn π^{lead}). Thus, there does not exist a pure-strategy equilibrium.

Corollary 13 *Under the demand system in* (2) *and complementary products* ($E > 0$)*,*

(i) $p^{\text{M}} < p^{\text{follow}} < p^{\text{II}} < p^{\text{I}} < \min\{p^{\text{DD}}, p^{\text{DDfollow}}\} < \max\{p^{\text{DD}}, p^{\text{DDfollow}}\} < p^{\text{DDlead}} < p^{\text{D}} < p^{\text{lead}}$,

(ii) $\pi^{\text{DDfollow}} < \pi^{\text{I}}$ *and* $\max\{\pi^{\text{follow}}, \pi^{\text{I}}\} < \pi^{\text{DDlead}} < \pi^{\text{DD}} < \pi^{\text{D}} < \pi^{\text{II}} < \pi^{\text{lead}} < \pi^{\text{M}}$,

(iii) $(\pi^{\text{DDlead}} + \pi^{\text{DDfollow}}) < (\pi^{\text{D}} + \pi^{\text{I}}) < 2\pi^{\text{DD}} < (\pi^{\text{lead}} + \pi^{\text{follow}}) < 2\pi^{\text{II}} < 2\pi^{\text{M}}$.

6. Conclusions

This paper fills the gap in the literature on strategic decentralization by considering the impact decision speed on the supply chains' strategic interaction and profitability. Decentralization and the associated speed of decision-making process affect the ability of the firm to respond to market signals. Oftentimes, decentralization compromises a supply chain's decision speed.

We show that while decentralization that makes the firms "too slow" will destroy the strategic benefits of commitment, decentralization that is "moderately slow" will enhance the strategic value of decentralization. This implies that, when firms compete, they may not want to maximize their decision speed.

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A. Proofs

Proof of Lemma 1. Supply chain i's problem:

$$\max_{p_i} p_i\, q_i(p_1, p_2) = \frac{-Bp_i^2 + p_i\big(A(B+E) - Ep_j\big)}{B^2 - E^2} \tag{24}$$

The best p_i in response to p_j, $j \neq i$, is

$$p_i(p_j) = \frac{A(B+E) - Ep_j}{2B}, \qquad i, j = 1, 2, \quad i \neq j. \tag{25}$$

The best responses lead to a symmetric Nash equilibrium. The equilibrium price is given in (4), and the resulting profit is given by (5). ∎

Proof of Lemma 2. Retailer i's problem:

$$\max_{p_i} (p_i - w_i)\, q_i(p_1, p_2) - f_i = \frac{(p_i - w_i)\big(A(B+E) - Bp_i - Ep_j\big)}{B^2 - E^2} - f_i \tag{26}$$

The best response function is

$$p_i(p_j; w_i) = \frac{A(B+E) - Ep_j + Bw_i}{2B}, \qquad i, j = 1, 2, \quad i \neq j. \tag{27}$$

The Nash equilibrium price is

$$p_i(w_1, w_2) = \frac{A(B+E)}{2B+E} + \frac{B(2Bw_i - Ew_j)}{4B^2 - E^2}, \qquad i, j = 1, 2, \quad i \neq j.$$

Manufacturer i chooses w_i to maximize the chain's profit:

$$\max_{w_i}\; p_i(w_1, w_2)\, q_i\big(p_1(w_1, w_2), p_2(w_1, w_2)\big) = p_i(w_1, w_2) \frac{A(B+E) - Bp_i(w_1, w_2) - Ep_j(w_1, w_2)}{B^2 - E^2}$$

It can be verified that the best response functions are linear and the equilibrium wholesale price is given in the lemma. Then, the equilibrium retail price is $p_i(w^{\mathrm{DD}}, w^{\mathrm{DD}}) \equiv p^{\mathrm{DD}}$ given in (8), and the equilibrium profit is given by (9). ∎

Proof of Lemma 3. The retailer's problem is the same as in (6). Hence, the best response is

$$p_1(p_2; w_1) = \frac{A(B+E) - Ep_2 + Bw_1}{2B}. \tag{28}$$

Chain 2's problem is the same as in (3). Hence, the best response is

$$p_2(p_1) = \frac{A(B+E) - Ep_1}{2B}.$$

The Nash equilibrium price is

$$p_1(w_1) = \frac{A(B+E)}{2B+E} + \frac{2B^2\, w_1}{4B^2 - E^2}$$
$$p_2(w_1) = \frac{A(B+E)}{2B+E} - \frac{BE\, w_1}{4B^2 - E^2}$$

In anticipation of this equilibrium result, chain 1's manufacturer will choose a wholesale price based on

$$\max_{w_1} \; p_1(w_1)\, q_1\big(p_1(w_1), p_2(w_1)\big)$$

The optimal wholesale price that chain 1 sets is given by w^{ID} in the lemma. Then the equilibrium retail prices are

$$p_1^{\mathrm{ID}} = p_1(w^{\mathrm{ID}}) = \frac{A(B+E)}{2B+E} + \frac{AE^2(B+E)}{2(2B+E)(2B^2-E^2)} = \frac{A}{2}\Big(1 + \frac{BE}{2B^2-E^2}\Big),$$
$$p_2^{\mathrm{ID}} = p_2(w^{\mathrm{ID}}) = \frac{A}{2}\Big(1 + \frac{E}{2B} - \frac{E^2}{2(2B^2-E^2)}\Big)$$

∎

Proof of Proposition 1. In the ID structure, the retail price competition is a Nash game, in which the integrated chain determines price by solving $\max_{p_2} p_2\, q_2(p_1, p_2)$, which is identical to the Stackelberg follower's problem. We only need to show that the decentralized manufacturer's problem coincides with the Stackelberg leader's problem.

The decentralized manufacturer determines the optimal w_1 by solving (10), which can be equivalently written as

$$\max_{w_1} \; p_1(w_1)\, q_1\big(p_1(w_1), p_2^*(p_1(w_1))\big), \tag{29}$$

where the function $p_2^*(p_1)$ is the integrated chain 2's best response function in response to p_1.

Notice that the problems in (11) and (29) are equivalent. Therefore, the decentralized manufacturer will choose a w_1 such that $p_1(w_1) = p_1^{\mathrm{lead}}$. Consequently, $p_2^*(p_1(w_1)) = p_2^{\mathrm{follow}}$.

Owen's note: More rigorously, we need to show the existence of w such that $p_1(w) = p^{\mathrm{lead}}$. ∎

Proof of Proposition 2. If the decentralized manufacturer determines w_1 after the integrated chain determines price, then clearly the integrated chain is the Stackelberg leader. As a follower, the decentralized manufacturer's optimal wholesale price is $w_1 = 0$, inducing its retailer to set a price that maximizes channel profit given the other chain's price decision.

Now consider the case when the decentralized manufacturer determines w_1 at the same time as the integrated chain decides p_2. We prove that $w_1 = 0$ and $p_2 = p_2^{\mathrm{lead}}$ is a Nash equilibrium. Clearly, the decentralized manufacturer's best response to $p_2 = p_2^{\mathrm{lead}}$ is $w_1 = 0$. Under $w_1 = 0$, retailer 1 behaves as an integrated chain, and decides p_1 after chain 2 makes decision. Thus, in response to $w_1 = 0$, chain 2 sets $p_2 = p_2^{\text{lead}}$. Therefore, although the game involves simultaneous move, the integrated chain has the leadership.

Owen's note: Better if prove uniqueness under some conditions. ∎

Proof of Proposition 3. In the final stage of the game, given w_1 and p_2, the decentralized retailer 1's problem is the same as in (6) and the optimal retail price is $p_1^*(w_1, p_2)$, which is expressed in (18).

In anticipation of retailer 1's response, the integrated chain 2 sets the retail price by solving

$$\max_{p_2} \; p_2 \, q_2\big(p_1^*(w_1, p_2), p_2\big) = p_2 \frac{A(B+E) - Bp_2 - E\frac{A(B+E) - Ep_2 + Bw_1}{2B}}{B^2 - E^2},$$

where we used (2) and (18). Thus, the integrated chain's optimal retail price (in response to w_1) is:

$$p_2^*(w_1) = \frac{A(2B-E)(B+E) - BEw_1}{2(2B^2 - E^2)} = p^{\text{lead}} - \frac{BE}{2(2B^2 - E^2)} w_1 \tag{30}$$

and the decentralized retailer's optimal retail price as a function of w_1 is:

$$\begin{aligned}
p_1^*(w_1, p_2^*(w_1)) &= \frac{A(B+E) - Ep_2^*(w_1) + Bw_1}{2B} \\
&= \frac{A}{2}\Big(1 + \frac{E}{2B} - \frac{E^2}{2(2B^2 - E^2)}\Big) + \frac{4B^2 - E^2}{4(2B^2 - E^2)} w_1 \\
&= p^{\text{follow}} + \frac{4B^2 - E^2}{4(2B^2 - E^2)} w_1
\end{aligned}$$

In the first stage, the decentralized manufacturer 1 sets w_1 to maximize its own chain's profit:

$$\begin{aligned}
&\max_{w_1} \; p_1^*(w_1, p_2^*(w_1)) \frac{A(B+E) - Bp_1^*(w_1, p_2^*(w_1)) - Ep_2^*(w_1)}{B^2 - E^2} \\
&= \frac{-B(4B^2 - E^2)(4B^2 - 3E^2)w_1^2 + 2AE^2(B+E)(4B^2 - 2BE - E^2)w_1}{16(B^2 - E^2)(2B^2 - E^2)^2} + \frac{A^2(B+E)(4B^2 - 2BE - E^2)^2}{16B(B-E)(2B^2 - E^2)^2}
\end{aligned}$$

The optimal wholesale price is

$$w^{\text{ID}'} = \frac{AE^2(B+E)(4B^2 - 2BE - E^2)}{B(4B^2 - E^2)(4B^2 - 3E^2)}.$$

Then, the equilibrium retail prices are

$$\begin{aligned}
p^{\text{I}} = p_2^*(w^{\text{ID}'}) &= \frac{A}{2}\Big(1 + \frac{E(8B^3 - 6BE^2 - E^3)}{(4B^2 - E^2)(4B^2 - 3E^2)}\Big) \\
p^{\text{D}} = p_1^*(w^{\text{ID}'}, p_2^*(w^{\text{ID}'})) &= \frac{A}{2}\Big(1 + \frac{E(2B^2 - E^2)}{B(4B^2 - 3E^2)}\Big)
\end{aligned}$$

and the equilibrium profits are

$$\pi^{\mathrm{D}} = \frac{A^2(B+E)(4B^2-2BE-E^2)^2}{4B(B-E)(4B^2-E^2)(4B^2-3E^2)} \tag{31}$$

$$\pi^{\mathrm{I}} = \frac{A^2(B+E)(2B^2-E^2)(8B^3-4B^2E-4BE^2+E^3)^2}{2B(B-E)(4B^2-E^2)^2(4B^2-3E^2)^2} \tag{32}$$

∎

Proof of Proposition 7. In the final stage of the game, decentralized retailer's problem is the same as in (6) and the optimal pricing for given w_1 and p_2 is $p_1^*(w_1, p_2)$ given in (18). It does not depend on w_2.

In anticipation of retailer 1's response, chain 2 sets the retail price by solving

$$\max_{p_2}\ (p_2 - w_2)\, q_2\big(p_1^*(w_1, p_2), p_2\big) = (p_2 - w_2)\frac{A(B+E) - Bp_2 - E\frac{A(B+E)-Ep_2+Bw_1}{2B}}{B^2-E^2}$$

where we used (2) and (18). Thus, the integrated chain's optimal retail price (in response to w_1) is:

$$p_2^*(w_1, w_2) = \frac{1}{2}\Big(A + w_2 + \frac{BE(A-w_1)}{2B^2-E^2}\Big) \tag{33}$$

and

$$p_1^*(w_1, p_2^*(w_1, w_2)) = ...$$

The decentralized manufacturer 1 sets w_1 to maximize its own chain's profit:

$$\max_{w_1}\ p_1^*(w_1, p_2^*(w_1, w_2))\frac{A(B+E) - Bp_1^*(w_1, p_2^*(w_1, w_2)) - Ep_2^*(w_1, w_2)}{B^2-E^2}$$

The optimal wholesale price is

$$w_1^*(w_2) = \frac{E^2\big[A(B+E)(4B^2-2BE-E^2) - E(2B^2-E^2)w_2\big]}{B(4B^2-E^2)(4B^2-3E^2)}.$$

Chain 2 sets wholesale price by solving

$$\max_{w_2}\ p_2^*(w_1^*(w_1), w_2))\frac{A(B+E) - Bp_2^*(w_1^*(w_2), w_2)) - Ep_1^*(w_1^*(w_2), p_2^*(w_1^*(w_1), w_2)))}{B^2-E^2}$$

The equilibrium wholesale prices are:

$$w_2^{\mathrm{DD}'} = \frac{AE^4(B+E)(8B^3-4B^2E-4BE^2+E^3)}{4(2B^2-E^2)^2(8B^4-8B^2E^2+E^4)}$$

$$w_1^{\mathrm{DD}'} = \frac{A(2B-E)E^2(B+E)(8B^3-6BE^2-E^3)}{4B(2B^2-E^2)(8B^4-8B^2E^2+E^4)}$$

The equilibrium retail prices are

$$p^{\text{DDlead}} \equiv p_2^{\text{DD}'} = \frac{1}{2}\Big(A + \frac{AE(4B^3 - 3BE^2)}{8B^4 - 8B^2E^2 + E^4}\Big)$$
$$p^{\text{DDfollow}} \equiv p_1^{\text{DD}'} = \frac{A(2B-E)^2(B+E)(2B+E)(8B^3 - 6BE^2 - E^3)}{8B(2B^2 - E^2)(8B^4 - 8B^2E^2 + E^4)}$$

The equilibrium profits are

$$\pi^{\text{DDlead}} \equiv \pi_2^{\text{DD}'} = \frac{A^2(B+E)(8B^3 - 4B^2E - 4BE^2 + E^3)^2}{16B(B-E)(2B^2 - E^2)(8B^4 - 8B^2E^2 + E^4)}$$
$$\pi^{\text{DDfollow}} \equiv \pi_1^{\text{DD}'} = \frac{A^2(2B-E)^3(B+E)(2B+E)(4B^2 - 3E^2)(8B^3 - 6BE^2 - E^3)^2}{64B(B-E)(2B^2 - E^2)^2(8B^4 - 8B^2E^2 + E^4)^2}$$

∎

Proof of Proposition 4. Compare $w^{\text{ID}'}$ with the wholesale price under DD structure in Lemma 2, we have

$$w^{\text{ID}'} = w^{\text{DD}}\Big(1 + \frac{2E^2(2B^2 - E^2)}{(4B^2 - E^2)(4B^2 - 3E^2)}\Big) > w^{\text{DD}}. \tag{34}$$

Compare $w^{\text{ID}'}$ with the wholesale price w^{ID} under ID structure with timing T1, we have

$$w^{\text{ID}'} = w^{\text{ID}}\Big(1 + \frac{E^2\big[(B-E)(8B^2 - 3E^2) + BE^2\big]}{(2B-E)^2(2B+E)(4B^2 - 3E^2)}\Big) > w^{\text{ID}}. \tag{35}$$

Using the expressions in (21)-(20), we can verify that $p^{\text{I}} < p^{\text{D}}$. Using (14)-(15), we have

$$p^{\text{follow}} = p^{\text{lead}}\Big(1 - \frac{E^2}{2B(2B-E)}\Big) < p^{\text{lead}}$$

That is, follower undercuts the leader.

We have

$$p^{\text{I}} = p^{\text{DD}}\Big(1 - \frac{E^5}{2B(4B^2 - E^2)(4B^2 - 3E^2)}\Big)$$

$$p^{\text{lead}} = p^{\text{DD}}\Big(1 + \frac{E^3}{4B(2B^2 - E^2)}\Big)$$

$$p^{\text{follow}} = p^{\text{II}}\Big(1 - \frac{E^3}{4B(2B^2 - E^2)}\Big)$$

Thus, if $E < 0$, we have the price ordering in the proposition.

$$\pi^{\text{M}} - \pi^{\text{I}} = \frac{A^2E^2\big[2(2B^2 - E^2)\big(16B^3(B^2 - E^2) + (3B-E)E^4\big) + BE^6\big]}{4B(B-E)(4B^2 - E^2)^2(4B^2 - 3E^2)^2} > 0$$

$$\pi^{\mathrm{I}} - \pi^{\mathrm{D}} = -\frac{A^2E^5(B+E)\big((B-E)(16B^2-5E^2)+BE^2\big)}{4B(B-E)(4B^2-E^2)^2(4B^2-3E^2)^2} \text{ sign depends on } E \text{ only}$$

$$\pi^{\mathrm{D}} - \pi^{\mathrm{DD}} = \frac{A^2E^8(B+E)}{4B(B-E)(4B^2-E^2)(4B^2-3E^2)(4B^2+2BE-E^2)^2} > 0$$

$$\pi^{\mathrm{follow}} - \pi^{\mathrm{lead}} = -\frac{A^2(4B-3E)E^3(B+E)}{16B(B-E)(2B^2-E^2)^2} \text{ sign depends on } E \text{ only}$$